# **Revision: Calculus (Topic 6)**

**Coursebook chapters: 16–20**

**1. Do not use a calculator to answer this question.**

For the function *f*(*x*) = *ax*3 + *bx*2 + 4*x* − 3, given that *f* ′(2) = 0 and *f* ″(2) = 10, find *a*, *b* ∈ ℝ.

*(accessible to students on the path to grade 3 or 4) [5 marks]*

**2.** Find the exact value of:



*(accessible to students on the path to grade 3 or 4) [4 marks]*

**3. Do not use a calculator to answer this question.**

Consider the function:



(a) Show that this can be written in the form *f*(*x*) = *xa* – 8*xb* + 12*xc* giving the values of the real numbers *a*, *b* and *c*.

(b) Find the equation of the normal to *f*(*x*) at the point *x* = 4.

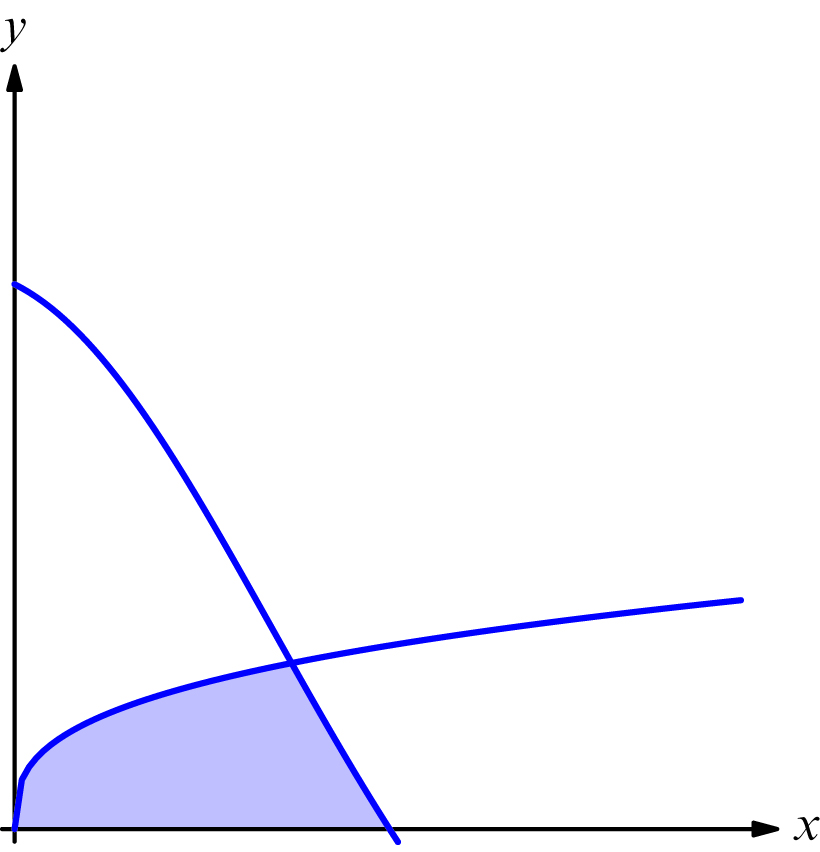
(c) The normal intersects the *x*-axis at the point *P* and the *y*-axis at the point *Q*.

(i) State the coordinates of *P* and *Q*.

(ii) Give the exact area of the triangle *POQ*.

*(accessible to students on the path to grade 3 or 4) [13 marks]*

**4.** The graph shows curves with equations *y* =  and *y* = – *x*. Find the area of the shaded region.



*(accessible to students on the path to grade 5 or 6) [6 marks]*

**5. Do not use a calculator to answer this question.**

Consider the curve *y* = 3*x*4 – 8*x*3 + 6*x*2 – 2.

(a) Find the coordinates of all stationary points.

(b) Classify these stationary points.

(c) Hence sketch the curve.

*(accessible to students on the path to grade 5 or 6) [12 marks]*

**6.** By using a suitable substitution or otherwise, find:



*(accessible to students on the path to grade 5 or 6) [6 marks]*

**7.** Differentiate the following:

(a) *y* = e2*x* tan2 3*x*

(b) *y* = 

*(accessible to students on the path to grade 5 or 6) [9 marks]*

**8.** A particle *P* moves in a straight line, passing the point *O* with speed 35 ms−1. At time *t* seconds after leaving *O* the acceleration *a* ms−2 is given by:

*a* = 6*t* − 22 0 ≤ *t* ≤ 5

(a) (i) Find an expression for the velocity at time *t*.

(ii) Find the times at which *P* is at rest.

(iii) Find the maximum speed of the particle in its 5 second journey.

(b) Find the total distance travelled by *P*.

*(accessible to students on the path to grade 5 or 6) [11 marks]*

**9. Do not use a calculator to answer this question.**

Consider the function *y* = 3*x*.

(a) (i) Show that = 3*x* ln 3.

(ii) Hence find .

(b) Use the substitution *u* = 3*x* to find the exact value of:



*(accessible to students on the path to grade 5 or 6) [12 marks]*

**10.** Find the coordinates of the stationary points of *x*2 + *y*2 – 3+ 20 = 0.

*(accessible to students on the path to grade 5 or 6) [9 marks]*

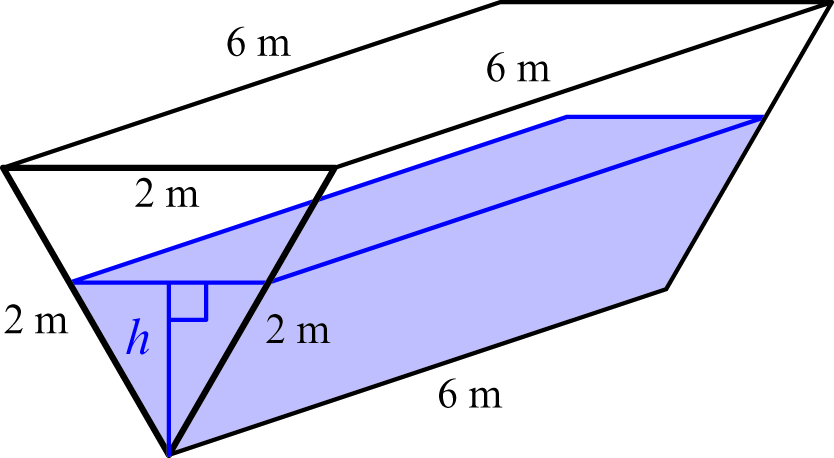
**11.** (a) Show that:

(2*x* – sin 2*x*) + *c*

(b) Hence find the exact value of the volume of the solid formed when *y* =  for 0 ≤ *x* ≤  is rotated 2π radians about the *x*-axis.

*(accessible to students on the path to grade 5 or 6) [12 marks]*

**12.** A water storage tank is in the shape of a triangular prism, as shown:



Water drips into the tank at a constant rate of 600 cm3s−1.

(a) Show that the volume of water in the tank (in m3) when the depth is *h* is given by *V* =.

(b) Find the rate of increase of the depth of water in the tank at the instant the tank is a quarter full.

*(accessible to students on the path to grade 5 or 6) [7 marks]*

**13.** (a) Show that:



(b) Find the exact value of:



*(accessible to students on the path to grade 7) [9 marks]*

**14. Do not use a calculator to answer this question.**

(a) Find *a*, *b* ∈ ℝ such that 11 + 10*x* – *x*2 = *a* – (*x* – *b*)2 for all *x*.

(b) Hence find *c* ∈ ℝ such that:



*(accessible to students on the path to grade 7) [8 marks]*