

Revision: Calculus (Topic 6)**Coursebook chapters: 16–20****1. Do not use a calculator to answer this question.**

For the function $f(x) = ax^3 + bx^2 + 4x - 3$, given that $f'(2) = 0$ and $f''(2) = 10$, find $a, b \in \mathbb{R}$.

(accessible to students on the path to grade 3 or 4) [5 marks]

2. Find the exact value of:

$$\int_{-\pi/3}^{\pi/2} \cos\left(\frac{1}{2}x\right) dx$$

(accessible to students on the path to grade 3 or 4) [4 marks]

3. Do not use a calculator to answer this question.

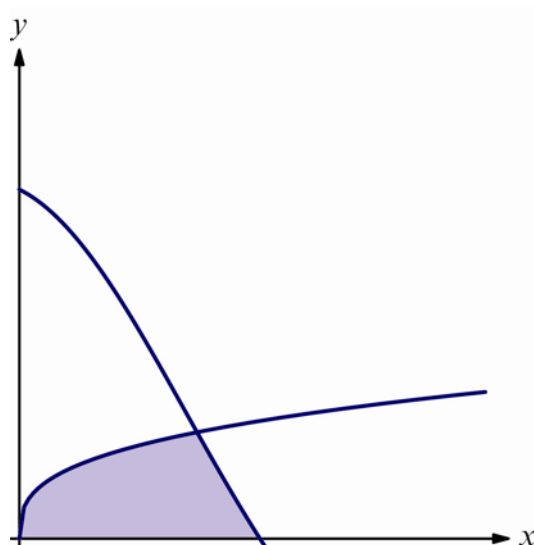
Consider the function:

$$f(x) = \frac{(x-2)(x-6)}{\sqrt{x}}$$

- (a) Show that this can be written in the form $f(x) = x^a - 8x^b + 12x^c$ giving the values of the real numbers a, b and c .
- (b) Find the equation of the normal to $f(x)$ at the point $x = 4$.
- (c) The normal intersects the x -axis at the point P and the y -axis at the point Q .
 - (i) State the coordinates of P and Q .
 - (ii) Give the exact area of the triangle POQ .

(accessible to students on the path to grade 3 or 4) [13 marks]

4. The graph shows curves with equations $y = \sqrt[3]{x}$ and $y = 3e^{-x^2} - x$. Find the area of the shaded region.



(accessible to students on the path to grade 5 or 6) [6 marks]

5. Do not use a calculator to answer this question.

Consider the curve $y = 3x^4 - 8x^3 + 6x^2 - 2$.

- Find the coordinates of all stationary points.
- Classify these stationary points.
- Hence sketch the curve.

(accessible to students on the path to grade 5 or 6) [12 marks]

6. By using a suitable substitution or otherwise, find:

$$\int \frac{4x}{x^2 - 9} dx$$

(accessible to students on the path to grade 5 or 6) [6 marks]

7. Differentiate the following:

(a) $y = e^{2x} \tan^2 3x$

(b) $y = \frac{\ln(1+x^2)}{1+x^2}$

(accessible to students on the path to grade 5 or 6) [9 marks]

8. A particle P moves in a straight line, passing the point O with speed 35 ms^{-1} . At time t seconds after leaving O the acceleration $a \text{ ms}^{-2}$ is given by:

$$a = 6t - 22 \quad 0 \leq t \leq 5$$

- (a) (i) Find an expression for the velocity at time t .
(ii) Find the times at which P is at rest.
(iii) Find the maximum speed of the particle in its 5 second journey.
- (b) Find the total distance travelled by P .

(accessible to students on the path to grade 5 or 6) [11 marks]

9. Do not use a calculator to answer this question.

Consider the function $y = 3^x$.

- (a) (i) Show that $\frac{dy}{dx} = 3^x \ln 3$.

- (ii) Hence find $\int y \, dx$.

- (b) Use the substitution $u = 3^x$ to find the exact value of:

$$\int_0^1 \frac{3^{2x}}{3^x + 1} \, dx$$

(accessible to students on the path to grade 5 or 6) [12 marks]

10. Find the coordinates of the stationary points of $x^2 + y^2 - 3 + 20 = 0$.

(accessible to students on the path to grade 5 or 6) [9 marks]

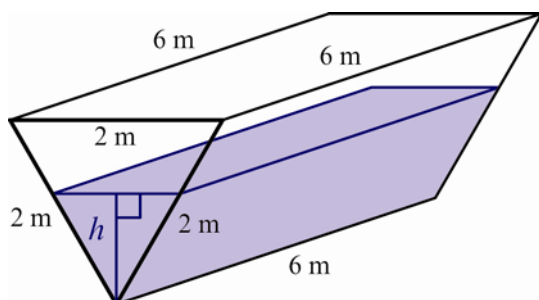
11. (a) Show that:

$$\int \sin^2 x \, dx = \frac{1}{4}(2x - \sin 2x) + c$$

- (b) Hence find the exact value of the volume of the solid formed when $y = \sqrt{x} \sin x$ for $0 \leq x \leq \frac{\pi}{2}$ is rotated 2π radians about the x -axis.

(accessible to students on the path to grade 5 or 6) [12 marks]

12. A water storage tank is in the shape of a triangular prism, as shown:



Water drips into the tank at a constant rate of $600 \text{ cm}^3 \text{ s}^{-1}$.

- (a) Show that the volume of water in the tank (in m^3) when the depth is h is given by $V = 2\sqrt{3}h^2$.
- (b) Find the rate of increase of the depth of water in the tank at the instant the tank is a quarter full.

(accessible to students on the path to grade 5 or 6) [7 marks]

13. (a) Show that:

$$\frac{d}{dx} \left(\arctan \frac{x}{3} \right) = \frac{3}{x^2 + 9}$$

- (b) Find the exact value of:

$$\int_{\sqrt{3}}^3 \arctan \frac{x}{3} dx$$

(accessible to students on the path to grade 7) [9 marks]

14. Do not use a calculator to answer this question.

- (a) Find $a, b \in \mathbb{R}$ such that $11 + 10x - x^2 = a - (x - b)^2$ for all x .
- (b) Hence find $c \in \mathbb{R}$ such that:

$$\int_5^c \frac{1}{\sqrt{11 + 10x - x^2}} dx = \frac{\pi}{6}$$

(accessible to students on the path to grade 7) [8 marks]