# Revision: Statistics and probability (Topic 5)

**Coursebook chapters: 21–24**

**1.** Find the mean and standard deviation of the data given by the following grouped frequency table:

|  |  |
| --- | --- |
| **Group** | **Frequency** |
| 6–12 | 26 |
| 12–17 | 18 |
| 17–22 | 45 |

*(accessible to students on the path to grade 3 or 4) [4 marks]*

**2.** A child looks for seashells lying on the beach. On average, there are 3 shells in every square metre of sand where she is searching. What is the probability that there are more than 20 in a 6 m2 area?

*(accessible to students on the path to grade 3 or 4) [4 marks]*

**3.** *X* is a continuous random variable with a pdf:

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(a) Given E(*X*) = 3, show that *k* = .

(b) Find the variance of *X*.

*(accessible to students on the path to grade 3 or 4) [6 marks]*

**4.** John takes the bus to work if it is raining or if he leaves home late; if he is on time and it is not raining, he walks to work. The probability that it rains on a given day is 0.3. The probability that he leaves home late is 0.6, independent of the weather.

(a) What is the probability that John walks to work?

(b) Given he took the bus, what is the probability that it was raining?

*(accessible to students on the path to grade 3 or 4) [6 marks]*

**5.** Measured salt levels in water samples taken from a lake are seen to follow a normal distribution with mean 0.3 and standard deviation 0.04. Two measurements are taken. Find the probability that:

(a) both measurements are greater than 0.36

(b) one measurement is greater than 0.4 and one is less than 0.4.

*(accessible to students on the path to grade 3 or 4) [6 marks]*

**6.** *X* is a continuous random variable with a pdf:



Determine the value of *a*.

*(accessible to students on the path to grade 5 or 6) [5 marks]*

**7.** Greig has a coin with a probability of returning 'Heads' when flipped.

After four coin-tosses, Greig has one 'Tails' and three 'Heads'.

(a) Find the probability, in terms of *p*, of flipping three or four 'Heads' out of four coin-tosses.

(b) What is the least value of *p* such that the probability of flipping three or four 'Heads' out of four coin-tosses is at least 5%?

*(accessible to students on the path to grade 5 or 6) [5 marks]*

**8.** Paul is playing a card game. On each hand he can score 0, 1 or 2 points, with probability  and  respectively.

(a) What is the expected score from a single hand?

(b) At the end of the fourth hand, Paul has scored exactly 3 points. What is the probability that he ever had a 2-point hand?

*(accessible to students on the path to grade 5 or 6) [7 marks]*

**9.** A factory produces tins of beans with masses normally distributed with mean 252 g. In order to label the tins as containing a minimum 250 g of product, the manufacturer must ensure that 99% of tins have at least this mass. What standard deviation is acceptable?

*(accessible to students on the path to grade 5 or 6) [5 marks]*

**10.**  Vesna and Stephen each played a game at a fairground.

Vesna's game involved shooting an air-rifle at moving targets. She can be expected to hit, on average, one target every 40 seconds. If she hit at least five targets in 2 minutes, she won a prize. There was no restriction on the number of shots she could take in the 2 minute period.

Stephen's game involved using a bow to shoot arrows at a target. He had a 0.7 probability of hitting on each shot (independent of previous results) and had 10 arrows. If he could hit the target eight or more times, he won a prize.

Given that exactly one of them won a prize, what is the probability that it was Vesna?

*(accessible to students on the path to grade 5 or 6) [7 marks]*

**11.** A particular breed of chicken produces eggs whose masses are known to follow a normal distribution with mean 50 g and standard deviation 4 g.

It is found that the probability of any egg with mass in excess of 60 g being 'double-yolked' is 10%.

Find the probability of seeing exactly 1 double-yolked egg in a random sample of 12 eggs.

*(accessible to students on the path to grade 5 or 6) [5 marks]*

**12.** *X* is a continuous random variable with a pdf:

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(a) Sketch the graph of *f*(*x*).

(b) Write down the range of possible values for *X*.

(c) Find an equation for *k* in terms of *a*.

*(accessible to students on the path to grade 7) [8 marks]*

**13.** *X* is a discrete random variable. For a given function *g*, expectation E(*g*(*X*)) is given by:

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Show that if *X* ~ B(*n*, *p*) then E(*ax*) = (1 *−* *p* + *pa*)*n*.

*(accessible to students on the path to grade 7) [5 marks]*

**14.**  is a continuous random variable with a pdf:

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(a) Sketch the graph of *y* = .

(b) Using your sketch, determine a lower bound for and an upper bound for *b*.

(c) Find the values of *a* and *b* if *b* – *a* = 1.8.

*(accessible to students on the path to grade 7) [8 marks]*