

## Chapter notes: 22 Probability

### Overview

As the ‘From another perspective’ box on page 705 highlights, probability is one of the most recent mathematical disciplines. In many countries it is not taught until the final year at school, so you may have students who have not encountered it before, as well as students who will be familiar with most of the sections A to D. If students are not familiar with these concepts, it will require around 12 hours of teaching time.

### Introductory problem

The introductory problem highlights the importance of understanding the sample space and conditional probabilities, as well as using precise language. The ‘Theory of knowledge issues’ point on page 740 invites students to think about the role of language in mathematics. The ‘Research explorer’ box on the same page refers to a game show involving cars and goats; there are many online simulations of the game. The worked solution is given at the end of the chapter, page 740; the idea being that students should be able to answer the question using the methods covered in the chapter.

### 22A Introduction to probability, p705

The ‘Research explorer’ box on page 706 mentions the ‘law of large numbers’. This states that the average of the results obtained from repeating an experiment a large number of times, nearly always tends to the theoretical probability. Students should be aware that, for example, repeatedly rolling a ‘six’ on a die is possible, although the probability tends to zero.

The ‘Research explorer’ box on page 707 refers to the Schrödinger’s Cat. In this thought experiment we cannot tell whether a cat in a box is dead or alive, so we could say that it is alive with certain probability.

### 22B Combined events and Venn diagrams, p711

The basic ideas about Venn diagrams are covered in Prior learning section J. This section mainly looks at the formula connecting the intersection and union of two sets:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The ‘Theory of knowledge issues’ box brings up again the point about the use of language in mathematics.

### 22C Tree diagrams and finding the intersection, p715

We use tree diagrams to introduce the concept of conditional probability; this is probability that depends on some previous information (such as burgers being one of the options for lunch).

*Hints for grade 7 questions:*

7. Denote the number of black disks by  $b$  and draw a tree diagram.

## 22D Independent events, p720

We define independent events as those having  $P(A|B) = P(A)$ . This corresponds to the intuitive notion of independence. The formula  $P(A \cap B) = P(A)P(B)$  can then be derived using the definition of conditional probability.

Many textbooks quote the latter formula as the definition of independent events. The ‘Theory of knowledge issues’ box asks students to consider the difference between a definition and a property. The distinction is not always clear; we can have several equivalent definitions. The answer to the last question in the ‘Theory of knowledge issues’ box is ‘No’: there are shapes other than circles that have constant width, one example being the British 50 pence coin.

## 22E Counting principles in probability, p722

This section is a good opportunity to revise chapter 1.

*Hints for grade 7 questions:*

6. (b) Arrange the backs and substitutes, then fit the forwards into the ‘gaps’.

## 22F Conditional probability, p724

This section continues the discussion of conditional probability from section 22C.

*Hints for grade 7 questions:*

5. Label  $P(A \cap B) = x$  and draw a Venn diagram.

## 22G Further Venn diagrams, p727

This section extends the work on Venn diagrams to include more than two sets.

The ‘Theory of knowledge issues’ box invites students to think about relative merits of using formulae and diagrammatic representations. Many people worry that visual arguments are not valid in mathematics, but this is not necessarily the case.

## 22H Bayes’ theorem, p732

Bayes’ theorem, with more than two events, is a new addition to the syllabus. The disease example may be a good one to use to introduce this chapter to students who have studied a lot of probability before.

The two ‘Theory of knowledge issues’ boxes (pages 733 and 735) encourage students to think about the function of proof in mathematics. The common perception is that its only purpose is to establish truth. However, the second box suggests that it can also be used to enhance understanding.

*Hints for grade 7 questions:*

11. Using a Venn diagram (with  $P(A \cap B)$  labelled  $x$ ) or a tree diagram (with  $P(A) = x$ ) is easier than trying to use the conditional probability formula.
12. This is the same as the blood test example in the main text.
13. You are looking for  $P(\text{fair}|\text{heads})$ . You can use the Bayes’ formula, but a tree diagram is probably easier.