

Self-assessment: 24 Continuous distributions

1. The amount of coffee dispensed by a machine follows normal distribution with mean 150 ml and standard deviation 5 ml.

- (a) Calculate the probability that the machine dispenses less than 142 ml of coffee.
- (b) Find the value of a if 20% of cups contain more than a ml of coffee.

(accessible to students on the path to grade 3 or 4) [5 marks]

2. A continuous random variable has probability density function given by:

$$f(x) = \begin{cases} k - x^2 & \text{for } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the value of k .
- (b) Calculate $P(X < 0.3)$.

(accessible to students on the path to grade 3 or 4) [5 marks]

3. It is known that the heights of a certain plant follow a normal distribution. In a sample of 200 plants, 32 are less than 45 cm tall and 50 are more than 88 cm tall. Estimate the mean and the standard deviation of the heights.

(accessible to students on the path to grade 5 or 6) [6 marks]

4. Random variable X has the probability density function given by:

$$f(x) = \begin{cases} \frac{4}{5}(3x - x^3) & \text{for } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

- (a) Calculate the expected value and the variance of X .

(accessible to students on the path to grade 3 or 4)

- (b) Find the mode of X .

(accessible to students on the path to grade 5 or 6)

- (c) (i) Show that the median of X satisfies $2m^4 - 12m^2 + 5 = 0$.

- (ii) Hence find the median of X .

(accessible to students on the path to grade 7)

[14 marks]