

Chapter notes: 23 Discrete probability distributions

Overview

The focus in this chapter is on predicting statistics when probabilities are known. Two common distributions – the binomial and the Poisson – are introduced. It needs approximately eight hours of teaching time.

Introductory problem

The introductory problem should get students thinking about the idea of expected average, and whether this is actually related to the most likely outcome. In this example, the expected average is infinite, but the most likely outcome is that you win £2. The ‘Theory of knowledge issues’ box at the end of the section (page 764) invites students to think about consequences of this. The worked solution is given at the end of the chapter, page 764; the idea being that students should be able to answer the question using the methods covered in the chapter.

23A Random variables, p743

There are no specific teacher notes for this section.

23B Expectation, median and variance of a discrete random variable, p747

The formulae for expectation and variance are given in the formula book. You can relate them to the formulae for calculating mean and variance for sets of data.

Do not dwell too much on the cumulative probabilities. However, the concept is needed for binomial and Poisson calculations on the calculator.

Students often find it difficult to understand why variance is such an important concept when it does not actually tell us anything directly about the random variable. As the ‘From another perspective’ box on page 748 explains, the variance is much neater mathematically. It hints at results from the Statistics option, in particular the important result that, when we add independent random variables, we add their variances; this is not true of standard deviations.

Hints for grade 7 questions:

9. (a) Probabilities can be found by drawing a tree diagram.

23C The binomial distribution, p751

The derivation of the formula for the binomial probabilities can be related to tree diagrams and counting principles. It is important to be able to use the formula, but most questions should be answered using the calculator. Worked Example 23.7 illustrates what needs to be written down when the calculator is used.

Hints for grade 7 questions:

16. This requires the use of the formula $P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$.

23D The Poisson distribution, p758

It is important that students understand the difference between the Poisson and binomial distributions and can select the correct one in a given context. The fact that $E(X) = \text{Var}(X)$ is sometimes tested by asking whether it would be reasonable to model given data by a Poisson distribution.

Hints for grade 7 questions:

- 16.** (c) We need to count the number of days when both are used, plus half the number of days on which one is used.
- 17.** (b) Use a calculator to create a table of probabilities.
- (d) We are looking for the smallest k such that $P(X \leq k) \geq 0.98$. This is best done by creating a list of probabilities on the calculator. See Calculator skills sheet 13.
- 18.** Use the formula.
- 19.** (a) Use the formula.